MAGNETIC MONOPOLES AS GAUGE PARTICLES?

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We present evidence for the following conjecture: when quantized, the magnetic monopole soliton solutions constructed by 't Hooft and Polyakov, as modified by Prasad, Sommerfield and Bogomolny, form a gauge triplet with the photon, corresponding to a Lagrangian similar to the original Georgi–Glashow one, but with magnetic replacing electric charge.

Gains in symmetry are usually important in physics. Introducing magnetic monopoles gains an unusual kind of symmetry: a "dual invariance" with respect to rotations through $\pi/2$ between the electric and magnetic directions of the Maxwell tensor, valid in vacuo, is preserved in the presence of matter. Dirac showed [1] that in quantum theory the possible electric and magnetic charges q and g must satisfy [‡1]

$$qg = 2\pi\hbar n, \quad n = 0, \pm 1, \pm 2, \ldots \tag{1}$$

This respects the dual symmetry

$$q \to g; \quad g \to -q . \tag{2}$$

Such an inverse relation between the strengths of charges is possibly related to the existence of strong and weak interactions in nature. There the relevant gauge groups H appear to be non-abelian in contrast to the abelian U(1) of the Maxwell theory. Analysis of the quantization condition analogous to condition (1) for a generalized non-abelian magnetic charge reveals that another group $H^{\vee}$, explicitly constructed out of H, plays a rôle [2]. It was suggested:

(A) that H monopoles behave as irreducible multiplets of $H^{\vee}$;

(B) that the field theory of H monopoles should have an $H^{\vee}$ gauge symmetry.

Since $(H^{\vee})^{\vee} = H$, there should be two "dual equivalent" field formulations of the same theory in which electric (Noether) and magnetic (topological) quantum numbers exchange rôles.

In two space-time dimensions one explicit example of the interchange of Noether and topological charges is known [3]. The Thirring model provides the quantum field theory of the solitons of the sine-Gordon model.

To substantiate conjectures (A) and (B) for monopoles, one should construct the analogous field theory for soliton monopoles, at least in the simplest case when H

(a) has lowest rank, namely one, and

(b) is self-dual: $H = H^{\vee}$.

The rank-one Lie groups are H = U(1), SO(3) or SU(2), but only U(1) is self-dual (by condition (1)). The simplest Lagrangian with U(1) (Dirac) monopoles occurring as solitons is the Georgi–Glashow model considered by 't Hooft and Polyakov [4], in the special limit introduced by Prasad and Sommerfield [5]. We dare to suggest that the dual quantum field theory of the monopole solitons is actually based upon exactly the same Lagrangian (with possibly some parameters changed). In the original Lagrangian, the heavy gauge particles carry the U(1) electric charge, which is a Noether charge, while the monopole solitons carry magnetic charge which is a topological charge. In the equivalent "dual" field theory the fundamental monopole fields, we conjecture, play the rôle of the heavy gauge particles, with the magnetic charge being now the Noether charge (and so related to the new SO(3) gauge coupling constant). Note that this situation, if true, differs from the sine-Gordon–Thirring model equivalence cited above in so far as the equivalent field theory has exactly the same form as the original

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[‡1] Contrary to common usage we are using rationalized units for both the electric and magnetic charges; i.e., the fields of point charges are $\boldsymbol{E} = q\boldsymbol{r}/4\pi r^3$ and $\boldsymbol{B} = g\boldsymbol{r}/4\pi r^3$.

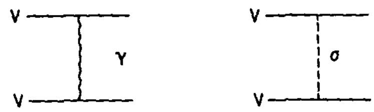

Fig. 1. Feynman graphs giving the long-range static potential between heavy gauge particles: a) photon exchange; b) Higgs meson exchange.

one – this would be the quantum field theoretical form of the "dual invariance" under the interchange of electric and magnetic charge, eq. (2).

We have unfortunately been unable to prove (or disprove) this conjecture, but we hope its potential interest justifies our explaining it in more detail and presenting some evidence in its favour: the classical properties of the monopole in the model, namely the spectrum, the mass and the long-range force two monopoles exert on each other are precisely related to the corresponding properties of the heavy gauge particle as our conjecture would predict.

The Georgi–Glashow Lagrangian has a "big" gauge group SO(3) and a real Higgs field in the triplet representation

$$\mathcal{L} = -\tfrac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \tfrac{1}{2} D_\mu \varphi_a D^\mu \varphi_a - \tfrac{1}{4}\lambda(\varphi_a\varphi_a - a^2)^2 . \quad (3)$$

Here

$$G_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + e_0 \epsilon_{abc} W_\mu^b W_\nu^c ,$$

$$D_{\mu,ab} = \partial_\mu \delta_{ab} + e_0 \epsilon_{acb} W_\mu^c .$$

In the ground state

$$\varphi_a \varphi_a = a^2 , \quad (4)$$

and the fact that φ must select a direction breaks the $G = \mathrm{SO}(3)$ symmetry down to $H = \mathrm{U}(1)$. The conventional particle content of the quantum field theory corresponding to eq. (3) is read off after the gauge

$$\varphi_a(x) = \delta_{a3}(a + \sigma(x)) , \quad (5)$$

is chosen. Then, if

$$W_\mu^3 = A_\mu , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad V_\mu = 2^{-1/2}(W_\mu^1 + \mathrm{i}W_\mu^2), \quad (6)$$

$\mathcal{L}$(eq. (3)) reads, apart from a divergence term,

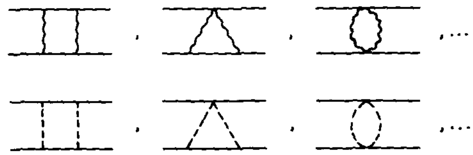

Fig. 2. Graphs not contributing to the static long range force.

$$\begin{aligned}\mathcal{L} = & -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2}(\partial_\mu V_\nu^+ - \partial_\nu V_\mu^+)(\partial^\mu V^\nu - \partial^\nu V^\mu) \\ & + 2\mathrm{i}e_0 F^{\mu\nu} V_\mu^+ V_\nu + \mathrm{i}e_0 A^\mu (V_\nu^+ \overleftrightarrow{\partial}_\mu V^\nu + V_\mu \partial_\nu V^{+\nu} - V_\mu^+ \partial_\nu V^\nu) \\ & + \tfrac{1}{4} e_0^2 (V_\mu^+ V_\nu - V_\nu^+ V_\mu)^2 - e_0^2 A^2 V_\mu^+ V^\mu \\ & + e_0^2 V_\mu^+ A^\mu V_\nu A^\nu + \tfrac{1}{2}(\partial_\mu \sigma)^2 + e_0^2 (a + \sigma)^2 V_\mu^+ V^\mu \\ & + \lambda a^2 \sigma^2 + \lambda a \sigma^3 + \tfrac{1}{4}\lambda \sigma^4 . \end{aligned} \quad (7)$$

The elementary quanta, with their electric charge, mass and spin, are:

the photon A_μ $(0, 0, \hbar)$,
the Higgs particle σ $(0, (2\lambda)^{1/2} a\hbar, 0)$,
the heavy gauge boson V_μ $(\pm q_0, M(q_0), \hbar)$,

where

$$q_0 = e_0 \hbar , \quad (8)$$

$$M(q_0) = a|q_0| = a|e_0|\hbar . \quad (9)$$

They correspond to classical particles in the limit

$$h \to 0; \quad q_0, a, (\lambda)^{1/2}\hbar \text{ held fixed} . \quad (10)$$

't Hooft and Polyakov showed that there are, in addition, soliton monopoles with magnetic charge

$$g = \pm g_0 ; \quad g_0 = 4\pi/e_0 = 4\pi\hbar/q_0 . \quad (11)$$

An important simplification occurs if we put $\lambda = 0$ but retain eq. (4) as a boundary condition to be satisfied at large distances [5]. The classical mass of the monopole state becomes precisely [6]

$$M(g) = a|g| = (4\pi\hbar/q_0^2) M(q_0) . \quad (12)$$

Note that the monopoles emerge as particles in a different classical limit, namely the usual loop expansion limit

$$\hbar \to 0 ; \quad e_0, a \text{ held fixed} . \quad (13)$$

Let us discuss the values of magnetic charge g allowed to elementary soliton solutions in general. It seems reasonable to require that all such solutions should be spherically symmetric in the sense of being covariant with respect to an angular momentum operator $\boldsymbol{J} = \boldsymbol{r} \wedge \boldsymbol{p} + \hbar \boldsymbol{t}$, where t_i are three fixed generators of the "big" group G satisfying $[t_i, t_j] = i\epsilon_{ijk} t_k$. Certainly all known soliton solutions satisfy this requirement, and we advance a reason below. If the little group $H = \mathrm{U}(1)$, g must satisfy [7] $\hat{\boldsymbol{r}}\cdot\boldsymbol{J} = \hat{\boldsymbol{r}}\cdot\boldsymbol{t}\hbar = -gQ/4\pi$, where Q is the electric charge operator (a matrix whose eigenvalues are the electric charges of the elementary fields).

For the theory under consideration, $\hat{\boldsymbol{r}}\cdot\boldsymbol{t}$ necessarily has eigenvalues 0, ±1 and Q eigenvalues 0, $\pm q_0$. Hence equating the smallest non-zero eigenvalues,

$$g q_0 = \pm 4\pi\hbar \, . \tag{14}$$

So the possible values for the magnetic charge of "elementary" states are only 0, $\pm g_0$ (this is sometimes known as the Guth–Weinberg theory [8]). The point we want to make about this is that this is precisely the same as the spectrum of values for the electric charge, assumed by the elementary fields in eq. (7), namely 0, $\pm q_0$. This is the first point of similarity between the soliton and elementary particle spectra.

The second point concerns the mass: our conjecture implies that we should be able to calculate the mass of the heavy gauge ($V^{\pm}$) particles regarding them as solitons of the field theory with the monopoles in the rôle of the gauge particles. Assuming $\lambda = 0$ and using eq. (12), we find, replacing q_0 by g_0, $M(q_0) = (4\pi\hbar/g_0^2)M(g_0)$. For consistency this should agree with eq. (9), which indeed it does. As a bonus we learn that the constant a is the same for both formulations of the theory since $a = M(g_0)/|g_0| = M(q_0)/|q_0|$. The two Lagrangians differ only in their coupling constants, $e_0 = (q_0/\hbar)$ and $g_0/\hbar$, respectively.

A third classical attribute of the monopole would be the intermonopole force. In an extremely ingenious calculation Manton [9] obtained the force from the instantaneous acceleration of two well-separated monopoles at rest and found an attraction $2g_0^2/4\pi r^2$ if the monopoles were oppositely charged but zero force if they were of like charge (at least to $O(1/r^2)$). The striking feature is that this is not the expected answer $\pm g_0^2/4\pi r^2$. According to our conjecture the $V^{\pm}$ particles should exert similar forces with q_0 replacing g_0. This can be checked, since the Born approximation for two particles moving slowly with respect to each other determines their static potential energy. The single photon exchange graph (fig. 1a) yields $\pm 4q_0^2 M^2/k^2$, indicating an attraction or repulsion $\pm q_0^2/4\pi r^2$ according to whether the charges are unlike or like. Since the Higgs particle has zero mass, its exchange gives a competing graph. The Yukawa coupling is read off from the term

$$e_0^2(a+\sigma)^2 V_\mu^+ V^\mu = \hbar^{-2}(M(q_0) + |q_0|\sigma)^2 V_\mu^+ V^\mu \, ,$$

in the Lagrangian (7). Even spin exchange being always attractive, the Higgs exchange graph (fig. 1b) gives $4q_0^2M^2/k^2$, or an additional attraction $q^2/4\pi r^2$, cancelling the repulsion between equally charged particles and doubling the attraction between oppositely charged ones. Thus two quite different calculations agree and support our conjecture. (The loop graphs in fig. 2 do not compete since they are $O(\hbar)$ compared to the ones in fig. 1 when the wave number – not the momentum – is held fixed, as it should be in calculating the potential.)

Thus not only does the monopole have a mass characteristic of a Higgs mechanism, it also possesses the characteristic Yukawa coupling.

So the classical properties of the monopole support our conjecture. So should the quantum properties but we do not know how to calculate them. Let us review the situation.

If the monopole is going to be a gauge boson, it should have a spin $1\hbar$. The presence of $\hbar$ emphasizes that it is a "zero point energy" quantum mechanical effect which is difficult to calculate by present methods and in particular not accessible to naïve semiclassical methods. Investigations up to now [10, 11] have concluded that the monopole is spinless, primarily because of the spherical symmetry of the classical solution. It seems to us, however, that the methods used cannot lead to conclusive result since they do not take into account operator ordering problems or, in the language of functional integration, the problems connected with non-linear canonical transformations of integration variables (for a discussion of this evergreen problem in a soliton context, see [12]). It is important to note that if the monopole has any fixed integer spin it must be spherically symmetric classically, since if not, there would be a tower of states with integrally spaced spins instead of just one value $n\hbar$. Thus the classical spherical symmetry requirement is indeed an "elementarity" requirement as supposed earlier.

Since the heavy gauge particle has a magnetic moment (as read off eq. (7)), $(q_0/M(q_0))\boldsymbol{J}_{\mathrm{Spin}} = a^{-1}\boldsymbol{J}_{\mathrm{Spin}}$, we predict that the monopole possesses a quantum mechanical electric dipole moment $a^{-1}J_{\text{Spin}} = (g/M(g))J_{\text{Spin}}$. Classically, the electric dipole moment is, of course, zero. (Note the gyromagnetic ratio is the same as for the Dirac electron.)

What about quantum corrections to the classical properties discussed earlier? Again we do not know but it is conceivable that the special properties discussed will survive because they are related to a symmetry. Thus we expect that, e.g., the mass formula (12) will remain valid when quantum corrections are taken into account, the parameters being interpreted as renormalized ones. When developing methods for a proper quantum treatment of monopoles, starting from the "electric" formulation of the theory, it is of paramount importance to take into account the existence of the monopoles from the start in order not to break the dual symmetry. Therefore we view with suspicion treatments based on the gauge (5), which cannot be chosen everywhere when monopoles are present.

Mention must be made of the dyon solutions [13]. As predicted by eq. (14) they have the two possible values of g, but, classically, any value of the electric charge between plus and minus infinity. Semi-classical quantization arguments indicate [14] that the charge is quantized in units of q_0 rather than $q_0/2$ which would be allowed by eq. (1), but would disagree with our conjecture, since states with magnetic charge $\pm g_0/2$ would be predicted contradicting eq. (14). The dyon states $(\pm q_0, \pm g_0)$ are self-dual, but the state with $|q| \geqslant 2q_0$ disagree with our conjecture, since they would imply the existence of states with multiple magnetic charge. We suspect that the state with $|q| \geqslant 2q_0$ will not survive a full quantum mechanical treatment.

Finally let us discuss the dyon mass formula [6]

$$M(q,g) = a(q^2+g^2)^{1/2}. \tag{15}$$

This formula has three remarkable features:

(1) it is universal in that it applies to *all* single particle states: dyons, monopoles, the photon, the Higgs boson and the heavy gauge particle;

(2) it guarantees the stability of the elementary states; since q and g are separately conserved we have by the triangle inequality

$$M(q_1+q_2, g_1+g_2) \leqslant M(q_1,g_1) + M(q_2,g_2);$$

(3) it exhibits a continuous symmetry under rotations in the (q,g) plane spontaneously broken by the allowed q,g values occurring in the model, maybe the Higgs particle (which has to be massless for eq. (15) to be valid) is the Goldstone boson for this peculiar sort of spontaneous symmetry breaking.

We have dared to present our speculations, as yet unproved, because we feel they do succeed in relating previously uncorrelated facts and would be, if true, of some importance for the further unveiling of the secrets of quantum gauge field theories.

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